CHAPTER 1

TIMSS 2027 Mathematics Framework

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Overview

Mathematics is a crucial academic subject for students, as it is an important building block for future learning and professional skill development. All students can benefit from developing an understanding of and facility with mathematics. In the early stages, mathematics learning supports children's emergent curiosity about how the world around them operates. Further on in their education, mathematics is essential in navigating daily life, and mastery of mathematics skills is relevant in many career fields.

Learning mathematics helps develop highly transferable skills, such as adaptability and problem solving, that are paramount in our ever-changing world. Mathematics can be used to bolster artistry and creativity, to make technical plans to execute a design, and to investigate patterns of the natural world.[1,2,3] Mathematics education enhances creative thought and invites students to understand the world in a logical, structured way, which in turn supports learning in other subjects.[4] Developing mathematical skills can position students for long-term success along vocational and academic career paths.[5,6] Mathematics supports essential everyday activities and decision-making like managing finances, estimating distances, or understanding data.[7]

The TIMSS 2027 Mathematics Framework builds on TIMSS's 32-year history of assessments and describes the mathematics measured in the TIMSS fourth-grade and eighth-grade assessments. This chapter presents the assessment frameworks for the two TIMSS 2027 mathematics assessments in the fourth and eighth grades. In general, the fourth- and eighth-grade mathematics frameworks are similar to those used in TIMSS 2023. Each cycle incorporates updates into the assessment frameworks to reflect the evolution of participating countries' curricula, standards, and frameworks. Sources informing these updates include the detailed reporting of the *TIMSS 2023 Encyclopedia*,[8] input from the TIMSS 2027 Science and Mathematics International Research Committee (SMIRC), and suggestions from the TIMSS 2027 National Research Coordinators.

The TIMSS 2027 mathematics assessments are designed such that a certain proportion of each area defined in the assessment frameworks is represented among the set of items included in the assessment, as described in the following sections. The TIMSS Mathematics Framework defines both the *content domain*, describing the topics to be assessed, and the *cognitive domain*, outlining the cognitive processes needed to solve mathematics items correctly. This document defines those domains that are reported as subscales to overall TIMSS mathematics achievement.

The primary purpose of this chapter is to outline the content and cognitive domains that form the foundation for international comparisons in the TIMSS Mathematics assessments. The TIMSS 2027 Mathematics Framework informs all item development for the TIMSS 2027 mathematics assessments at fourth and eighth grade. Each item in the TIMSS mathematics assessments is designed to target only one content domain and one cognitive domain. The frameworks also discuss key aspects in mathematics education and assessment that influence decisions about the assessment content and specifications.

TIMSS Mathematics Content Domains

TIMSS assesses student knowledge in mathematics subject matter via the definition of the content domains at each grade. The coverage and definition of the mathematics content domains differ for the fourth and eighth grades, reflecting the mathematics widely taught at each grade and the development of topics covered across international mathematics curricula. Exhibit 1.1 shows the target percentage of item score points in the assessment for each content domain for the TIMSS 2027 fourth- and eighth- grade assessments.

Exhibit 1.1: Target Percentages of the TIMSS 2027 Mathematics Assessment Devoted to Content Domains at the Fourth and Eighth Grades

Fourth Grade

Content Domains	Percentages
Number	50%
Measurement and Geometry	30%
Data	20%

Eighth Grade

Content Domains	Percentages
Number	30%
Algebra	30%
Geometry and Measurement	20%
Data and Probability	20%

Each content domain consists of at least one *topic area*, and each topic area, in turn, includes several *topics*. The framework for each grade has 20 topics comprising approximately 5 percent of the assessment overall. Accordingly, each topic receives approximately equal weight in score points across the items developed to measure them at each grade of the mathematics assessments.

In the fourth-grade mathematics assessment, there is more emphasis on topics in the number content domain than in either of the other two domains, reflecting similar curricular emphases

internationally. Moving to the eighth grade, algebra is added as a content domain to match the mathematics curricula at that grade level internationally. The introductory algebraic topics assessed at the fourth grade (sometimes referred to as pre-algebra) are included in the number content domain. Both measurement and geometry are included in both grades, but at the fourth grade, measurement is the primary focus of the domain, with only a preliminary understanding of geometric shapes required. In contrast, the eighth-grade geometry and measurement domain emphasize purely geometric topics. The fourth-grade data domain focuses on reading, representing, and interpreting data in a defined set of representations, whereas at the eighth grade, the data and probability domain emphasizes drawing conclusions from data, basic statistics, and the fundamentals of probability.

TIMSS Mathematics Cognitive Domains

In understanding mathematics, students need to know the mathematics content being assessed and must also draw on a range of cognitive skills. Numerous theories map the progression of students' cognition through childhood development broadly (e.g., Piaget, 1970) and and conceptual frameworks articulate objectives for learning (e.g., Krathwohl, 2002; Bloom, 1956).[9,10,11] Generally, the cognitive skills understood to underlie mastery of subject matter follow the progression from developing a knowledge base, to engaging that knowledge in unfamiliar contexts, and to making judgments or justifying arguments.[12,13]

In the TIMSS assessments, these sets of cognitive skills are referred to as cognitive domains and three domains have been identified: *knowing*, *applying*, and *reasoning*. The first domain, knowing, covers the recollection of facts or concepts and the execution of routine procedures. The applying domain focuses on applying factual and procedural knowledge and conceptual understanding in a range of situations. The reasoning domain involves the logical, systematic thinking students need to use to justify solutions to problems, make inferences, and understand the connections across mathematical concepts. TIMSS cognitive domains can be related to Bloom's taxonomy of educational objectives and its later revision.[14]

Briefly, remembering and understanding processes map to knowing; applying remains applying; and analyzing, evaluating, and creating (which refers to synthesizing knowledge and skills to generate something new) can be found in the reasoning domain. These cognitive processes are understood to build on each other and are frequently depicted as a pyramid of increasingly more challenging processes. Across the three cognitive domains of TIMSS mathematics, items requiring the 1) knowing, 2) applying, and 3) reasoning also present as, on average, increasingly difficult sets of tasks.

Exhibit 1.2 shows the target percentage of item score points in the assessment for each of the three cognitive domains at the fourth and eighth grades. Reflecting the development in skills and differences in the ages of students, the balance of score points differs between these grades.

Exhibit 1.2: Target Percentages of the TIMSS 2027 Mathematics Assessment Devoted to Cognitive Domains at the Fourth and Eighth Grades

Cognitive Domains	Fourth Grade	Eighth Grade
Knowing	40%	35%
Applying	40%	40%
Reasoning	40%	25%

Students exercise knowing, applying, and reasoning to varying extents when engaging in mathematical thinking. Identifying these skills is crucial in developing an assessment like TIMSS 2027, ensuring that the survey covers the appropriate range of cognitive skills across the content domains. Each TIMSS mathematics item is classified according to the primary cognitive process students access when responding correctly to the item. Content and cognitive skills are cross-classified; each item belongs to one content and one cognitive domain.

Problem Solving and Problem Contexts in TIMSS Mathematics

Problem solving is an overarching process of TIMSS mathematics carried out across the content domains. In general, "problem solving refers to cognitive processing directed at achieving a goal when the problem solver does not initially know a solution method."[15] Specifically for mathematics, problem solving can take place in a real-world context or in purely mathematical terms.[16]

Mathematical problems set in a real-world context can be addressed using a modeling cycle: First, the problem is translated from the real world into mathematical representations; then, mathematical knowledge, procedures, and reasoning are applied to those representations to produce a mathematical solution or explanation; finally, the solution or explanation is translated back to the real world and interpreted and validated.[17,18] If the original problem has not been solved, the cycle might need to be repeated with revised assumptions.

Within the scope of the TIMSS mathematics assessments, the processes involved in solving a mathematics problem are described in the TIMSS cognitive domains, where each item targets a single process defined therein. Every mathematics item in TIMSS is assigned one cognitive area within the three cognitive domains: knowing, applying, or reasoning. Individual items in TIMSS cannot prompt students to engage in the entire modeling cycle, but the collection of items included in TIMSS covers the range of cognitive skills in which students engage while moving through a full modeling cycle. Any single cognitive skill involved in the process of solving a problem can be included in any content domain by targeting the appropriate cognitive area within one of these cognitive domains. For instance, the cognitive domain *applying* defines the cognitive area *formulate*, which includes determining appropriate operations and instructions to address a problem; or the cognitive domain *knowing* defines the cognitive area *compute*, which includes using (known) algorithmic procedures to find a result.

In TIMSS items and in problem solving generally, the extent of familiarity with the problem context can contribute to the complexity of cognitive engagement required to successfully navigate all or part of the modeling cycle. Contexts can be immediately familiar or can require

some decoding, and the complexity of the decoding effort depends on the intended audience for the problem. For example, an item asking fourth-grade students to make the largest possible even number using a set of number cards would likely require substantial cognitive processing for that grade level to achieve the goal. Thus, the item poses a more complex problem for these students and would engage higher-order cognitive skills. The same item might not constitute a problem for students in eighth grade since the cognitive processes that they might need to draw on are simpler, possibly more automatic recall of relevant number facts.

For TIMSS mathematics, approximately 85 percent of items will be situated in a context; most of these will be suitable for problem solving by students at the appropriate grade level. Nonetheless, items without a real-world context can still be suitable for problem solving if they are non-routine for the student attempting them. The remaining 15 percent of the items in TIMSS mathematics will be presented without context, such that the possible effects of reading load do not affect students' ability to display their skills and knowledge.

Among the items situated within real-world contexts in TIMSS are Problem-Solving and Inquiry tasks (PSIs), offering extended measurement of problem solving by including multiple steps of problem solving and higher-order cognitive skills across several items situated within more complex scenarios. PSIs comprise independent items that are still individually assigned to one content and cognitive domain per item as defined in this framework. However, the PSIs establish a way to assess these domains more deeply and authentically, relying on the shared context across items to guide students through multiple stages of the modeling cycle throughout the task. This prepares students to engage with more complex scenarios and contexts than could be achieved in a single TIMSS item. Further information on the characteristics of PSIs can be found in the TIMSS 2027 Guidelines for Developing PSIs.[19]

TIMSS 2027 Mathematics Content Domains—Fourth Grade

Three major content domains define the mathematics content for the TIMSS Mathematics fourth-grade assessment: number, measurement and geometry, and data. Whole numbers are the predominant component of the number domain; students at the fourth grade compute with whole numbers of reasonable size. Fourth-grade students work with familiar objects and understand the relationships between shapes and sizes. Students at the fourth grade manipulate data to read and create basic data displays and make comparisons between different representations of the same data or between different datasets.

Number

The number content domain consists of three topic areas such that the 50 percent of the assessment devoted to number is apportioned as follows:

- Whole numbers (25%)
- Expressions, simple equations, and relationships (15%)
- Fractions and decimals (10%)

Learning about whole numbers provides the foundation of mathematics in primary school. Introductory algebraic concepts are also part of the TIMSS assessment at the fourth grade, including understanding the use of unknowns in simple equations and an initial understanding

of variables and relationships between quantities. However, because quantities and measures of object properties often do not come in whole numbers, it is also important for students to understand fractions and decimals.

Whole Numbers

1. Connect representations (i.e., words, symbols, and models including number lines), compare numbers up to 6 digits, and understand place value.
2. Add and subtract up to 4-digit numbers.
3. Multiply (up to 3-digit by 1-digit and 2-digit by 2-digit numbers) and divide (up to 3-digit by 1-digit numbers), including with a remainder.
4. Understand and use odd and even numbers, multiples and factors of numbers, round numbers (up to the nearest powers of 10), and make estimates.
5. Combine two or more properties of numbers (e.g., place value, odd/even) or arithmetic operations (e.g., double and add 5).

Expressions, Simple Equations, and Relationships

1. Find the missing number or operation in a number sentence (e.g., 17 + ■ = 29).
2. Match or write expressions or number sentences to represent problem situations that may involve unknowns.
3. Match, describe, or use relationships between numbers in a well-defined pattern (e.g., describe the relationship between adjacent terms of a sequence and generate whole numbers given a rule).

Fractions and Decimals

1. Describe a fraction as part of a whole or collection; connect different representations of fractions (i.e., words, numbers, and models); compare fractions including with different denominators; add and subtract simple fractions with the same denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100.
2. Connect different representations of decimals (i.e., words, numbers, and models); compare decimals and relate decimals to fractions; add and subtract decimals (up to two decimal places).

Measurement and Geometry

The two topic areas in measurement and geometry are as follows:

- Measurement (15%)
- Geometry (15%)

In primary school, measurement—the process of quantifying attributes of objects and phenomena—introduces performing basic calculations with length, mass, volume, and time, and using a ruler to measure length (standardized 10-centimeter onscreen ruler with millimeter gradation, which includes rotation and drag functionality). Spatial sense is integral to the study of geometry, and students analyze geometric relationships and use these relationships to draw conclusions about a variety of two-dimensional shapes and three-dimensional objects.

Measurement

1. Measure, estimate, add, and subtract lengths (e.g., millimeters, centimeters, meters, kilometers).
2. Add and subtract mass (e.g., grams and kilograms), volume (e.g., milliliters and liters), and time (e.g., minutes and hours); select appropriate types and sizes of units and read scales.
3. Determine perimeters of polygons, areas of rectangles, areas of shapes covered with squares or partial squares, and volumes filled with cubes.

Geometry

1. Recognize and draw parallel and perpendicular lines, right angles, and angles smaller or larger than a right angle; compare the relative size of angles.
2. Use elementary properties, including line symmetry, to describe and create common two-dimensional shapes (i.e., circles, triangles, quadrilaterals, and other polygons).
3. Use elementary properties to describe three-dimensional objects (i.e., cubes, rectangular solids, cones, cylinders, and spheres) and how they relate to their two-dimensional representations.

Data

The data content domain consists of two topic areas:

- Single dataset (15%)
- Multiple datasets (5%)

The widespread availability of data in today's information society has resulted in various visual displays of quantitative information, also commonly referred to as (data) visualizations. Thus, students need to understand that graphs and charts help organize information or categories and provide a way to compare data. At the fourth grade, students primarily interact with representations of single datasets, but they also begin to work with data from one or more sources.

Single Datasets

1. Read data from tables, pictographs, bar graphs, line graphs, and pie charts.
2. Create or complete tables, pictographs, bar graphs, line graphs, and pie charts.
3. Compare different representations of the same dataset.

Multiple Datasets

1. Combine or compare representations of two or more datasets.

TIMSS 2027 Mathematics Content Domains—Eighth Grade

Four content domains define the mathematics content for the TIMSS Mathematics eighth-grade assessment: number, algebra, geometry and measurement, and data and probability. Building on the number content domain at the fourth grade, eighth-grade students develop proficiency with more advanced whole number concepts and procedures and extend their mathematical understanding of rational numbers presented in various forms (integers, fractions, and decimals). With the introduction of algebra, students engage with patterns and relationships more formally with variables and functions. Extending their understanding of shapes and measures assessed at the fourth grade, eighth-grade students analyze the properties of a variety of two-dimensional shapes and three-dimensional objects and calculate perimeters, areas, and volumes. Eighth-grade students can read and extract meaning from a variety of displays and visualizations, begin to understand basic statistics, and start to work with probability.

Calculator Use at the Eighth Grade

At the eighth grade, students will be permitted to use the TIMSS on-screen calculator that is available for every item in the eighth-grade assessment. This calculator includes the four basic operations (+, –, ×, ÷), a square root key, and the negative sign. Students will not be permitted to bring their own calculators. The calculator executes one calculation at a time and does not include more advanced features such as graphing. Overall, the mathematics items are developed to be calculator-neutral and do not have advantages or disadvantage for students whether or not they use calculators. A notable exception is the (very few) items that require taking a square root, mainly in applications of the Pythagorean Theorem.

Number

At the eighth grade, the 30 percent of the assessment devoted to number consists of three topic areas:

- Integers (10%)
- Fractions and decimals (10%)
- Proportions, ratios, and percentages (10%)

In eighth grade, students need to compute with integers as well as fractions and decimals, and understand quantities represented in different forms. A single rational number can be represented with many different written symbols, and it is important that students can translate these representations to recognize the distinctions among interpretations of rational numbers, convert between them, and reason with them. This includes the facility to apply proportions, ratios, and percentages to whole numbers.

Integers

1. Recognize and use properties of numbers and operations; find and use multiples and factors, recognize prime numbers, and evaluate positive integer powers of numbers and square roots resulting in integers.

   Note: In other content domains (e.g., geometry), square roots may involve integers beyond perfect squares.

2. Add and subtract positive and negative numbers, including through movement and position on a number line or using various models (e.g., thermometers, losses and gains).

Fractions and Decimals

1. Using various models and representations, compare fractions and decimals, identify equivalent fractions and decimals, and round decimals.
2. Add, subtract, and multiply with fractions and decimals, and divide fractions and decimals by a whole number.

Proportions, Ratios, and Percentages

1. Determine proportions and ratios of quantities (e.g., rates, scales on maps, recipes).
2. Apply or find percentages; convert between percentages and fractions or decimals.

Algebra

The 30 percent of the assessment devoted to algebra is comprised of two topic areas:

- Expressions, operations, and equations (20%)
- Relationships and functions (10%)

The introduction of algebra is a major milestone in mathematics education—using algebraic models and expressing relationships algebraically, rearranging formulas, and substituting values into formulas. Conceptual understanding of such models and relationships can extend to linear and simple nonlinear functions to describe what will happen to one variable when a related variable changes.

Expressions, Operations, and Equations

1. Find the value of an expression or a formula given the values of the variables.
2. Simplify algebraic expressions involving sums, products, differences, and positive integer powers; compare expressions to decide if they are equivalent.
3. Write expressions, equations, or inequalities to represent problem situations.
4. Solve (or validate potential solutions to) linear equations; solve simple linear inequalities and simple simultaneous linear equations in two variables, including validating values as solutions.

Relationships and Functions

1. Interpret, relate, and generate representations of linear functions in tables, graphs, or words; recognize properties of linear functions, including slope and intercepts; generalize linear patterns or sequences using words or algebraic expressions.
2. Interpret, relate, and generate representations of simple nonlinear functions (e.g., quadratic) in tables, graphs, or words; and generalize nonlinear patterns or sequences using words or algebraic expressions.

Geometry and Measurement

The geometry and measurement content domain at the eighth grade consists of one topic area:

- Geometry and Measurement (20%)

At the eighth grade, students can relate and integrate representations of three-dimensional objects and apply that understanding to composites of known shapes. Students are able to apply their understanding of geometric relationships to plotting on the Cartesian plane. They can explain geometric relationships, such as congruence, similarity, and the Pythagorean Theorem.

Geometry and Measurement

1. Recognize and draw types of angles and pairs of lines and use the relationships between angles on lines and in geometric figures, including those involving the measures of angles and line segments; read and plot points in the Cartesian plane.
2. Recognize two-dimensional shapes and use their geometric properties (e.g., sums of interior angles of triangles and quadrilaterals, properties of isosceles triangles), including calculating length and area, and use the Pythagorean Theorem.

    Note: Two-dimensional shapes include (or can be composed of) circles; scalene, isosceles, equilateral, and right-angled triangles; trapezoids, parallelograms, rectangles, rhombuses, and other quadrilaterals; as well as other polygons including pentagons, hexagons, octagons, and decagons.
3. Determine the results of geometric transformations (i.e., translations, reflections, and rotations) in the plane; recognize and use properties of congruent and similar triangles and rectangles.
4. Recognize three-dimensional objects and use their properties to calculate surface area and volume; relate three-dimensional objects with their two-dimensional representations.

    Note: Three-dimensional objects include (or can be composed of) prisms, pyramids, cones, cylinders, and spheres.

Data and Probability

The data and probability content domain contains two topic areas:

- Data (15%)
- Probability (5%)

Increasingly, students encounter data in formats beyond the more traditional forms of data display (e.g., bar graphs, line graphs, pie graphs, pictographs)—information is being supplemented by an array of new graphic forms (e.g., infographics). Thus, students must extend their understanding of how to read, understand, organize, and represent data to being familiar with the statistics underlying data distributions and how these relate to the shape of data graphs. Students build on knowledge of organizing and representing data to make predictions and projections beyond what is represented. Students in eighth grade also have an initial grasp of some probability-related concepts.

Data

1. Organize and represent data in histograms, dot plots, scatter plots, clustered and stacked bar charts, and infographics, in addition to representations included at fourth grade (i.e., tables, pictographs, bar graphs, line graphs, and pie charts).
2. Interpret data from one or more sources, including making inferences, interpolating and extrapolating, and modeling (e.g., trends, predictions, productivity).
3. Summarize data distributions; calculate, use, or interpret mean and median; recognize the effect of spread and outliers.

Probability

1. Determine theoretical probability based on proportions of favorable outcomes (e.g., rolling a fair die or drawing marbles of a particular color from a bag); estimate empirical probability based on experimental outcomes.

TIMSS 2027 Mathematics Cognitive Domains—Fourth and Eighth Grades

For the fourth- and eighth-grade TIMSS assessments, each content domain includes items developed to address the three cognitive domains: knowing, applying, and reasoning. The following sections further describe the thinking processes that define these cognitive domains.

Each cognitive domain consists of a set of cognitive areas reflecting the cognitive processes defining that domain, elaborated by a brief description. Each TIMSS item is designed to target exactly one of these cognitive areas to ensure a range of coverage within each cognitive domain. There are no specified targets for the percentages of score points for each cognitive area across the TIMSS assessments.

Knowing

Facility in factual, conceptual, and procedural knowledge of mathematics forms the basis for applying mathematics or reasoning about mathematical situations.[20] Facts encompass the knowledge that provides the basic language of mathematics and the essential mathematical concepts and properties foundational for mathematical thought. Students access knowledge of relationships and representations, conventions and properties of numbers, symbolic representations, and spatial relations to engage in purposeful mathematical thinking. Procedures such as classifying or recognizing calculations and algorithms form the foundation of mathematics needed for solving problems encountered in daily life. The four cognitive areas included in knowing, form the foundation of mathematical fluency at both grades assessed in TIMSS.

Recall	Recall definitions, terminology, number properties, units of measurement, geometric properties, and notation (e.g., $a \times b = ab$, $a + a + a = 3a$).
Identify	Identify numbers, expressions, quantities, and shapes. Recognize when entities are mathematically equivalent. Read information from graphs, tables, texts, or other sources.
Order	Order and classify numbers, expressions, quantities, and shapes by common properties.
Compute	Compute arithmetic operations with whole numbers, fractions, decimals, and integers using algorithmic procedures. Carry out straightforward algebraic manipulation.

Applying

From a solid basis of knowledge, students can plan and execute to solve non-routine problems. As part of the problem-solving process, students may need to formulate the problem in mathematical terms, implementing a strategy to apply mathematical knowledge. Students will need to select suitable operations, strategies, and tools for solving problems. Ultimately, representing ideas or solutions forms the core of mathematical thinking and communication, and the ability to create representations is fundamental to success in the subject. The three cognitive areas of the applying domain involve the application of mathematics in a range of situations.

Formulate	Determine efficient/appropriate operations, instructions, programs, strategies, and sequences thereof for solving problems.
Implement	Implement suitable strategies, operations, or tools to produce solutions to problems.
Represent	Represent data in tables or graphs; create equations, inequalities, geometric figures, or diagrams that model problem situations; and generate equivalent representations for a given mathematical entity or relationship.

Reasoning

Reasoning mathematically involves logical, systematic thinking through observing and finding patterns and relationships and making conjectures. It also involves making logical deductions based on specific assumptions and rules and justifying results. Evidence of reasoning processes can be found in explaining or justifying a solution method, making valid inferences based on information and evidence, or generalizing mathematical relationships. Each of the four cognitive skills listed in the reasoning domain may be drawn on when solving problems in less familiar contexts or when integrating several concepts or strategies. These skills support the basis of computational thinking (decomposition, pattern recognition, abstraction, and algorithm development).

Analyze	Decompose a problem; evaluate, describe, or use relationships among numbers, expressions, quantities, and shapes.
Integrate	Link different elements of knowledge, related representations, and procedures.
Generalize	Make statements that represent relationships in more general and more widely applicable terms.
Justify	Provide mathematical arguments to support a strategy or solution.

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